Workshop 4

Value at Risk

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CCFEA Workshop on Computational Finance

Risk measures

- A risk measure will compute

  "how bad things can get"

- It should summarize all relevant information of the riskiness into one number
- The most popular measure by far is the Value-at-Risk (VaR)
- Other approaches include the EVaR, CVaR, expected short fall, etc

Coherent measures

Let X and Y denote financial losses. A *coherent* measure of risk $\rho(X)$ should satisfy the following axioms:

1. Subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$
2. Monotonicity: $X \leq Y$ for every scenario implies $\rho(X) \leq \rho(Y)$
3. Positive homogeneity: $\rho(\alpha X) = \alpha\rho(X)$ for all $\alpha > 0$
4. Translation invariance: $\rho(X+r) = \rho(X) + r$ for all $r > 0$

Value at risk

- Value at risk is defined as

$$VaR_p(X) = -\inf\{x|\mathsf{P}[X \leq x \cdot r] > p\}$$

- Intuitively VaR_p is telling us: "With probability $1-p$ we expect our losses to be less than VaR_p
- For normally distributed X we have

$$VaR_p(X) = -(\mu_X + \Phi^{-1}(p) \cdot \sigma_X)$$

- VaR_p is not a coherent measure of risk: It does not satisfy the subadditivity axiom

The max coherent measure

scen	X_1	X_2	$X_1 + X_2$	$2 \times X_1$	$X_1 + 1$
1	1	0	1	2	2
2	2	0	2	4	3
3	3	1	4	6	4
4	2	2	4	4	3
5	1	3	4	2	2
6	0	2	2	0	1
7	0	1	1	0	1
ρ	3	3	4	6	4

VaR complications

To compute the VaR we need the inverse cumulative function of the losses, the normal is usually assumed

- Fat tails and skewness
- Options and other nonlinearities
- Large number of securities

VaR computation

- Parametric: A density for the losses is assumed, normal, Student-t etc
- Variance-Covariance: The Delta/Gamma of the portfolio are also used to correct for nonlinearities
- Simulation based: Scenarios are generated and sorted. Very time consuming for exotic contracts that have to be recomputed.
- Combinations: Control variate, importance sampling or stratified sampling
- Principal components: PC analysis reduces the number of factors. Only works if instruments are very correlated

VaR extensions

- Economic VaR for risk neutral measure Q

$$EVaR_p(X) = -\inf\{x|\mathsf{Q}[X \leq x \cdot r] > p\}$$

- CVaR or "tail conditional expectation" or TailVaR

$$TCE_p(X) = -\mathsf{E}\{X/r|X/r \leq -VaR_p(X)\}$$

- Worst conditional expectation (a coherent measure)

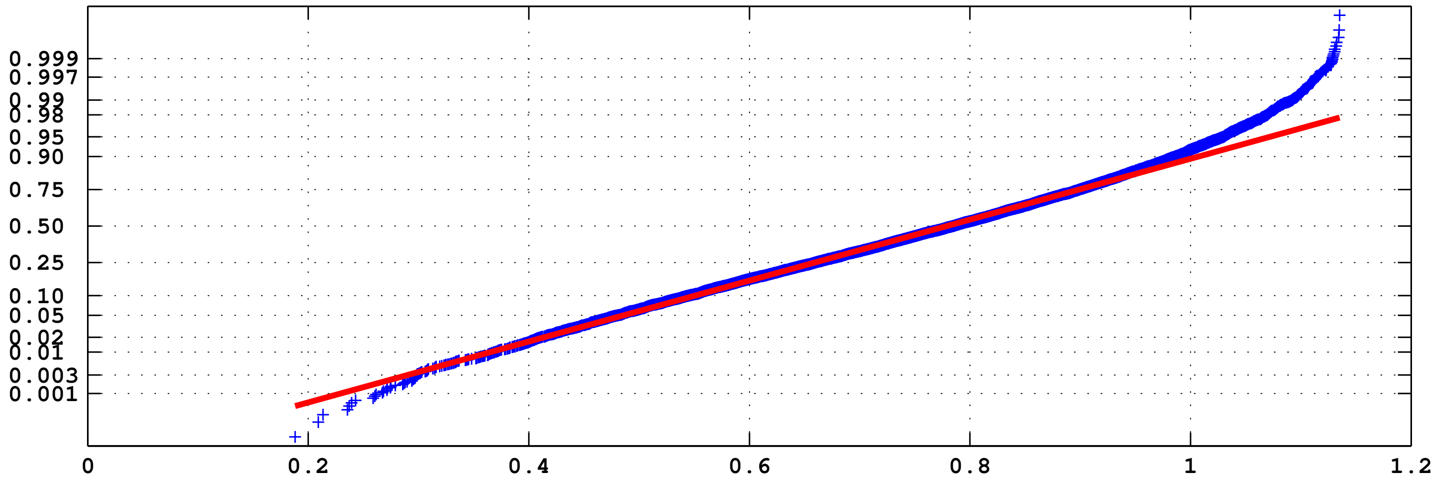
$$WCE_p(X) = -\inf\{\mathsf{E}[X/r|A]|\mathsf{P}[A] > p\}$$

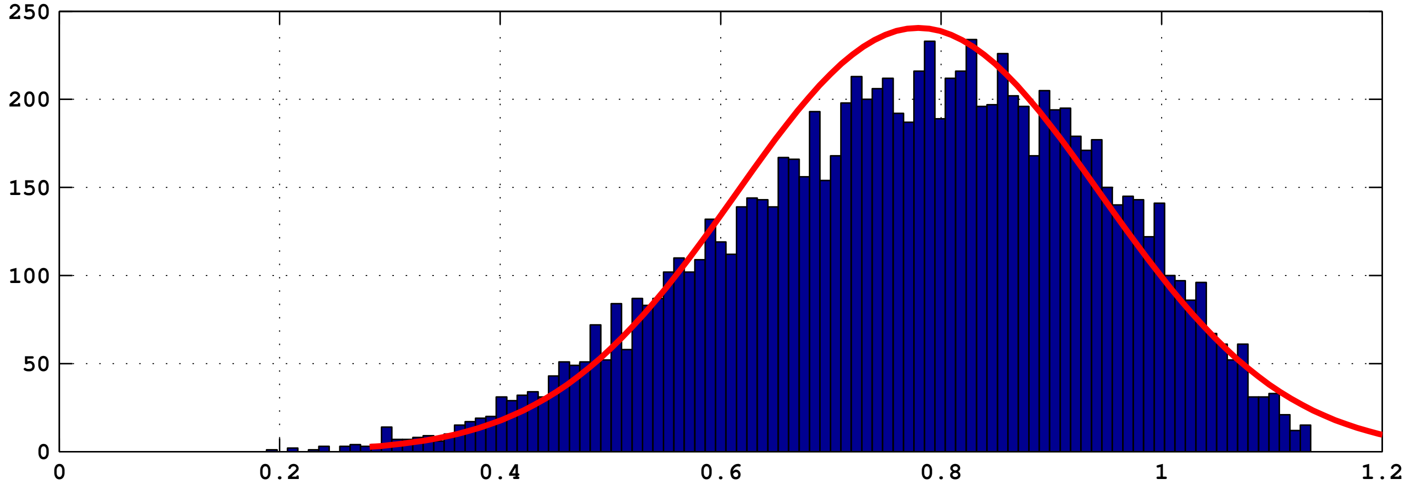
VaR computation

Computation of Monte-Carlo VaR and parametric VaR. Initial portfolio value .7738. 30,000 simulations.

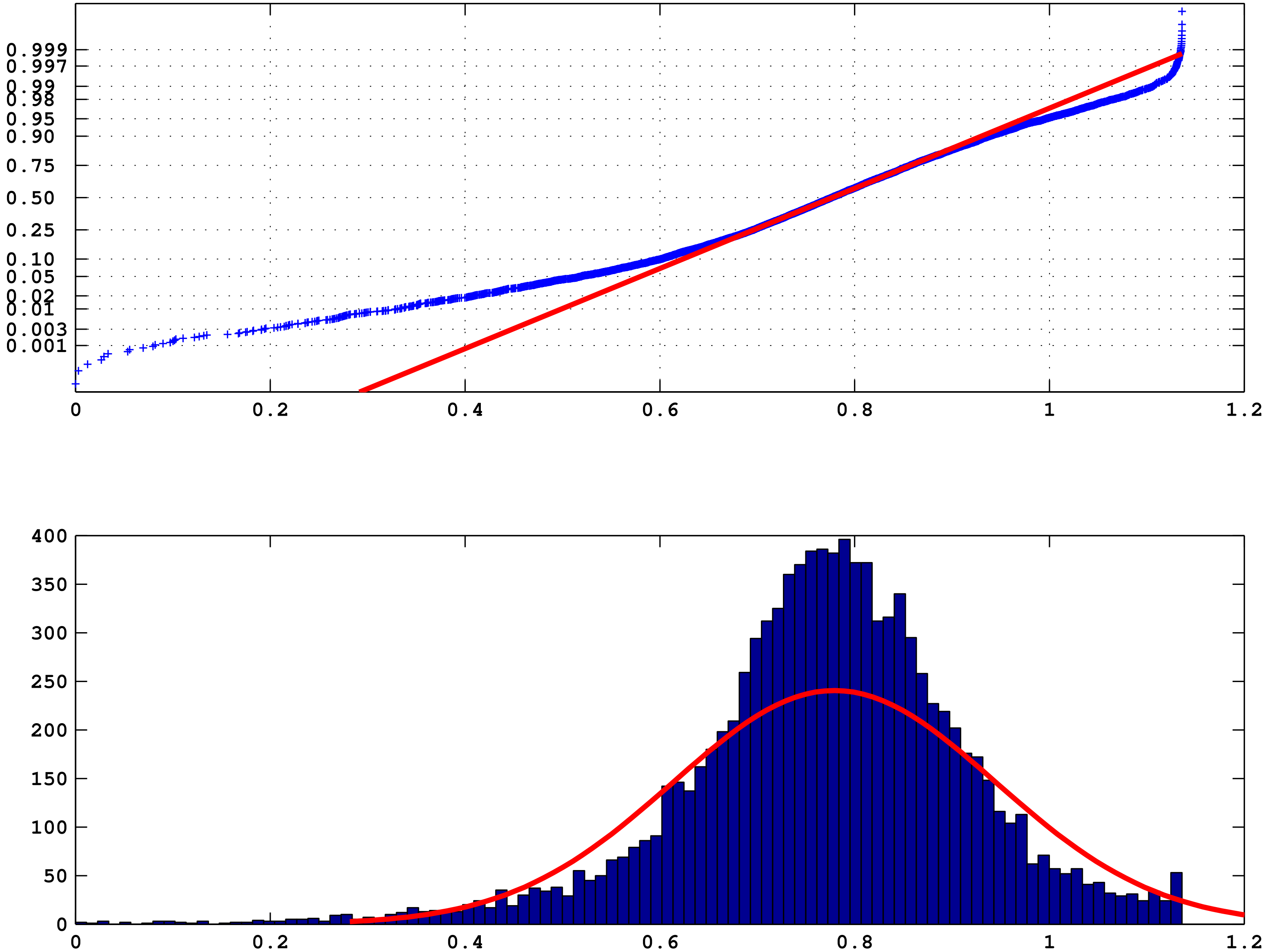
		MC-N	Δ	$\Delta - \Gamma$
Normal	5%	.4730	.5064	.5085
	1%	.3617	.3934	.3954
Stu-T	5%	.5257		
	1%	.3210		

Simulated VaR (Normal)





Simulated VaR (Student-t)



Principal components

- Say that the portfolio assets $\{S_i\}_{i=1}^{K}$ have the *factor* representation

$$\frac{dS_i}{S_i} = a_{i,0} + \sum_{j=1}^{N} a_{i,j} \frac{dF_{i,j}}{F_{i,j}} + u_i$$

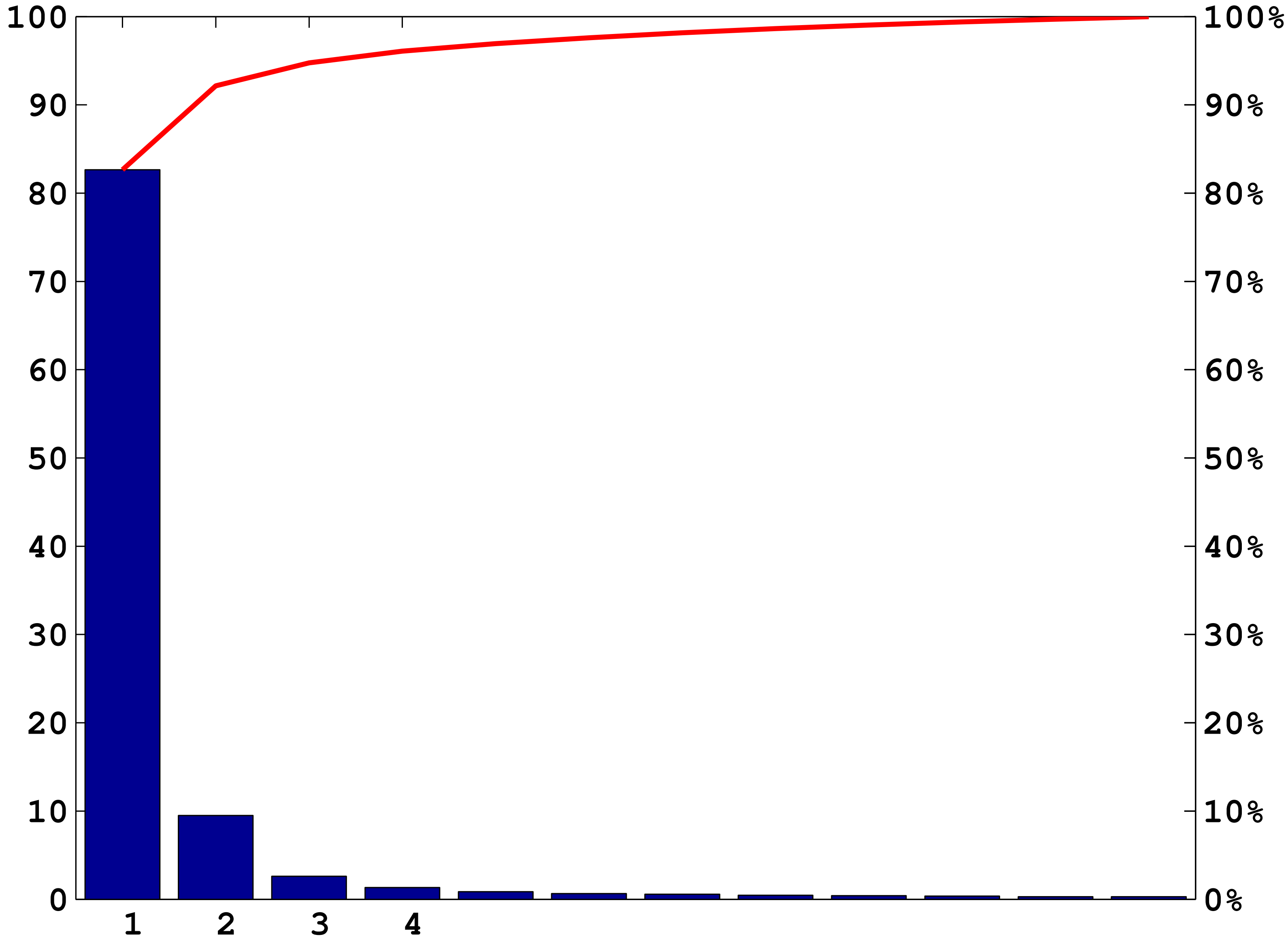
  for $N \ll K$ with the factors being independent processes

- Principal component analysis can extract this information from the data and isolate the factors that are responsible for most of the variability

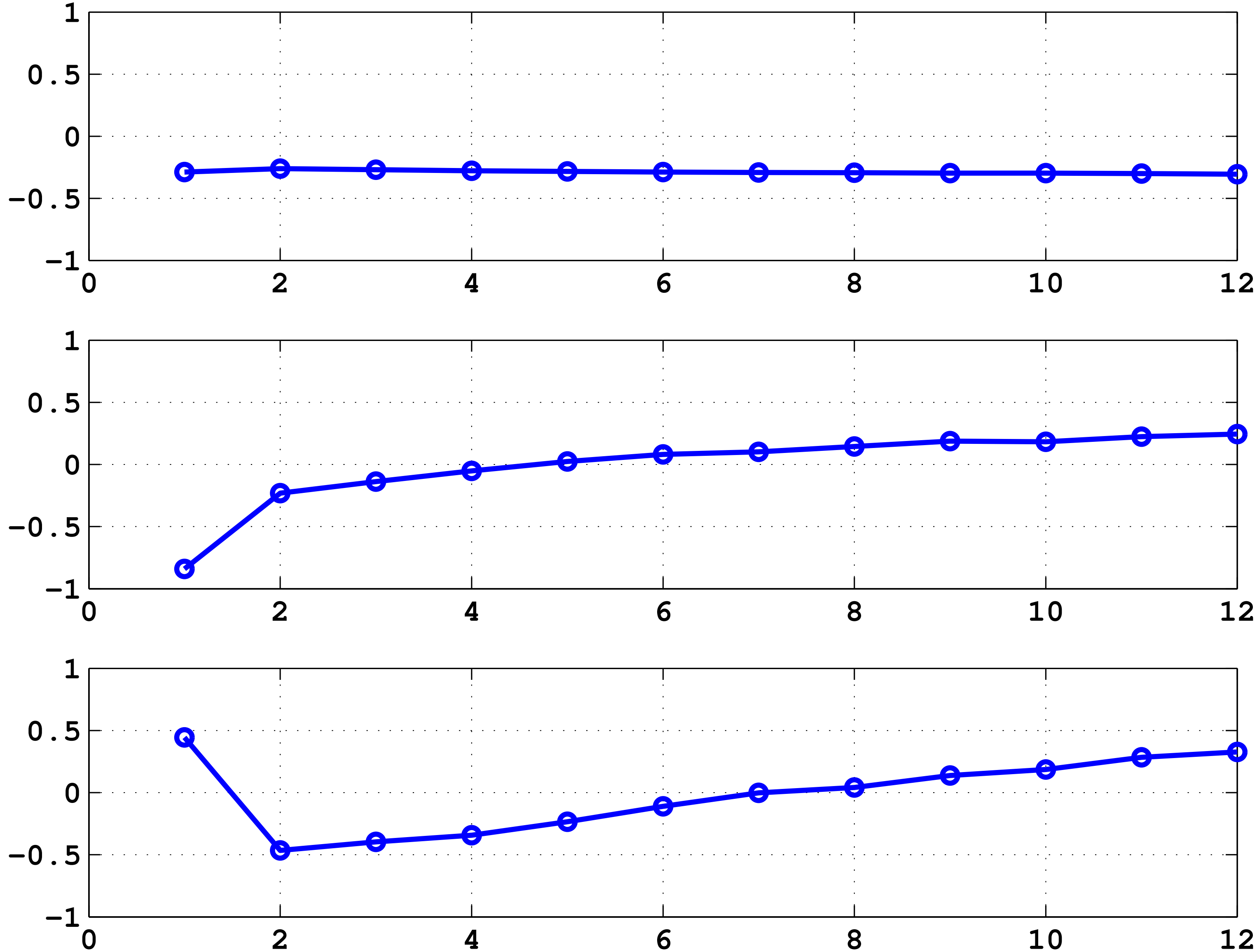
Principal components

- Data Y are in a $T \times K$ matrix, and can be represented as $Y = \mu + \sigma \cdot U$ where $\sigma' \cdot \sigma = \Sigma$ is the covariance matrix and U are uncorrelated random variables
- The eigenvalue/vector decomposition of Σ allows on to write
$$\Sigma = V \cdot \Lambda \cdot V^{-1}$$
- The eigenvalues Λ will show the relative importance of the corresponding eigenvectors
- Example: for interest rates three factors capture more than 95% of the variability. Factors are “shifts”, “twists” and “curvature”.

PC example: variances



PC example: factor loadings



PC example: fitted rates